Quenching of Hadron Spectra due to the Collisional Energy Loss of Partons in the Quark-Gluon Plasma

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We estimate the energy loss distribution and investigate the quenching of hadron spectra in ultrarelativistic heavy-ion collisions due to the collisional energy loss of energetic partons from hard parton collisions in the initial stage.

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In the initial stage of ultrarelativistic heavy-ion collisions energetic partons are produced from hard collisions between the partons of the nuclei. Receiving a large transverse momentum, these partons will propagate through the fireball which might consist of a quark-gluon phase for a transitional period of a few fm/c. These high-energy partons will manifest themselves as jets leaving the fireball. Owing to the interaction of the hard partons with the fireball these partons will lose energy. Hence jet quenching will result. The amount of quenching might depend on the state of matter of the fireball, i.e., quark-gluon plasma (QGP) or a hot hadron gas, respectively. Therefore jet quenching has been proposed as a possible signature for the QGP formation [1].

In order to use jet quenching as a signature for the quark-gluon plasma, the energy loss of hard partons in the QGP has to be determined. First the energy loss due to collisional scattering was estimated [2–6]. Using the Hard-Thermal-Loop (HTL) resummed perturbative QCD at finite temperature [7], the collisional energy loss of a heavy quark could be derived in a systematic way [8–11]. From these results also an estimate for the collisional energy loss of energetic gluons and light quarks could be derived [12], which was rederived later using the Leontovich relation [13,14]. Later also the energy loss due to multiple gluon radiation (bremsstrahlung) was estimated and shown to be the dominant process. For a review on the radiative energy loss see Ref. [15]. Recently, it has also been shown [16] that for a moderate value of the parton energy there is a net reduction in the parton energy loss induced by multiple scattering due to a partial cancellation between stimulated emission and thermal absorption. This can cause a reduction of the quenching factor due to radiative processes.

Unfortunately, jets, requiring very large initial parton energies, are rare events, which are difficult to observe. However, quenching of hard partons will also affect hadron spectra at high transverse momenta $p_\perp$. Indeed, first results from RHIC have indicated a suppression of high-$p_\perp$ spectra [17]. The consequences of jet quenching on hadron spectra have been calculated, for example, in Refs. [18,19]. Here only the radiative energy loss has been taken into account. The purpose of the present paper is to estimate the quenching of hadron spectra due to the collisional energy loss of partons in the QGP. As we will see, this contribution can be of the same order as the radiative quenching.

We will follow the investigations by Baier et al. [18] and Müller [19], using the collisional instead of the radiative parton energy loss. Following Ref. [18] the $p_\perp$ distribution is given by the convolution of the transverse momentum distribution in elementary hadron-hadron collisions, evaluated at a shifted value $p_\perp + \epsilon$, with the probability distribution, $D(\epsilon)$, in the energy ϵ, lost by the partons to the medium by collisions, as

$$\begin{aligned}\frac{d\sigma^{\rm med}}{d^2p_\perp} &= \int d\epsilon\, D(\epsilon)\,\frac{d\sigma^{\rm vac}(p_\perp+\epsilon)}{d^2p_\perp} = \int d\epsilon\, D(\epsilon)\,\frac{d\sigma^{\rm vac}}{d^2p_\perp} + \int d\epsilon\, D(\epsilon)\,\epsilon\,\frac{d}{dp_\perp}\frac{d\sigma^{\rm vac}}{d^2p_\perp} + \cdots\cdots \\ &= \frac{d\sigma^{\rm vac}}{d^2p_\perp} + \Delta E\cdot\frac{d}{dp_\perp}\frac{d\sigma^{\rm vac}}{d^2p_\perp} = \frac{d\sigma^{\rm vac}(p_\perp+\Delta E)}{d^2p_\perp} = Q(p_\perp)\frac{d\sigma^{\rm vac}(p_\perp)}{d^2p_\perp}. \qquad (1)\end{aligned}$$

Here $Q(p_\perp)$ is suppression factor due to the medium and the total energy loss by partons in the medium is

$$\Delta E = \int \epsilon\, D(\epsilon)\, d\epsilon\,. \qquad (2)$$

We need to calculate the probability distribution, $D(\epsilon)$, that a parton loses the energy, ϵ, due to the elastic collisions in the medium. This requires the evolution of the energy distribution of a particle undergoing Brownian motion. The

operative equation for the Brownian motion of a test particle can be obtained from the Boltzmann equation, whose covariant form can be written as

$$p^\mu \partial_\mu D(x,p) = C\{D\}\,, \tag{3}$$

where p is the momentum of the test particle, $C\{D\}$ is the collision term and $D(x,p)$ is the distribution due to the motion of the particle. If we assume a uniform plasma, the Boltzmann equation becomes

$$\frac{\partial D}{\partial t} = \frac{C\{D\}}{E} = \left(\frac{\partial D}{\partial t}\right)_{\rm coll} \quad . \tag{4}$$

We intend to consider only the elastic collisions of the test parton with other partons in the plasma. The rate of collisions $w(p,k)$ is given by

$$w(p,k) = \sum_{j=q,\bar{q},g} w^j(p,k)\,, \tag{5}$$

where w^j represents the collision rate of a test parton i with other partons, j, in the plasma. The expression for w^j can be written as

$$w^j(p,k) = \gamma_j \int \frac{d^3q}{(2\pi)^3} f_j(q) v_{\rm rel} \sigma^j\,, \tag{6}$$

where γ_j is the degeneracy factor, $v_{\rm rel}$ is the relative velocity between the test particle and other participating partons j from the background, and σ^j is the associated cross section. Due to this scattering the momentum of the test particle changes from p to $p-k$. Then the collision term on the right-hand side of (4) can be written as

$$\left(\frac{\partial D}{\partial t}\right)_{\rm coll} = \int d^3k \, \left[w(p+k,k)D(p+k) - w(p,k)D(p)\right] \ . \tag{7}$$

where the collision term has two contributions. The first one is the gain term where the transition rate $w(p+k,k)$ represents the rate that a particle with momentum $\vec{p}+\vec{k}$ loses momentum $\vec{k}$ due to the reaction with the medium. The second term represents the loss due to the scattering of a particle with momentum $\vec{p}$.

Now under the Landau approximation, i.e., most of the quark and gluon scattering is soft which implies that the function $w(p,p')$ is sharply peaked at $p \approx p'$, one can expand the first term on the right-hand side of (7) by a Taylor series as

$$w(p+k,k)D(p+k) \approx w(p,k)D(p) + \mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}}(wD) + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial p_j}(wD) + \cdots\cdots . \tag{8}$$

Combining (4),(7) and (8), the Fokker-Planck equation

$$\frac{\partial D}{\partial t} = \int d^3k \left[\mathbf{k}\cdot\frac{\partial}{\partial \mathbf{p}} + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial p_j}\right](wD) \tag{9}$$

is obtained, which describes the equation of motion for the distribution function of fluctuating macroscopic variables. We consider, for simplicity, the one dimensional problem, for which (9) can be written as

$$\frac{\partial D}{\partial t} = \frac{\partial}{\partial p}\left[\mathcal{T}_1(p)D\right] + \frac{\partial^2}{\partial p^2}\left[\mathcal{T}_2(p)D\right]. \tag{10}$$

Here the transport coefficients are given as

$$\begin{aligned} \mathcal{T}_1(p) &= \int d^3k\, w(p,k)\, k = \frac{\langle \delta p\rangle}{\delta t} = \langle F\rangle\,, \\ \mathcal{T}_2(p) &= \frac{1}{2}\int d^3k\, w(p,k)\, k^2 = \frac{\langle (\delta p)^2\rangle}{\delta t} \ . \end{aligned} \tag{11}$$

Now the work done by the drag force, $\mathcal{T}_1$, acting on a test particle is

$$-dE = \langle F \rangle \cdot dL = \mathcal{T}_1(p) \cdot dL \ , \tag{12}$$

which can be related to the energy loss [9,12] of a particle as

$$-\frac{dE}{dL} = \mathcal{T}_1(p) \approx p\,\mathcal{A} \ , \tag{13}$$

where $\mathcal{A}$ is the drag coefficient, which is almost independent of momentum p [3,20]. The drag coefficient is a very important quantity containing the dynamics of elastic collisions. In principle, $\mathcal{A}(p,t)$, may be determined from the kinetic theory formulation of QCD through the application of the fluctuation-dissipation theorem [21], but that is indeed a difficult problem. As discussed in Refs. [3,20,22], the drag coefficient is expected to be largely determined by the properties of the bath and not so much by the nature of the test particle. Then one can use the average of the drag coefficient given as

$$\langle \mathcal{A}(p,t) \rangle \ = \ \mathcal{A}(t) \ = \ \left\langle -\frac{1}{p}\,\frac{dE}{dL} \right\rangle \ . \tag{14}$$

The quantity $\mathcal{T}_2$ can be related to the diffusion coefficient in the following way:

$$\mathcal{T}_2(p) = \frac{\langle (\delta p)^2 \rangle}{\delta t} = p\mathcal{A}p \approx \mathcal{A}T^2 = \mathcal{D}_F \ , \tag{15}$$

where we have approximated p by the temperature T of the system and the drag by using the Einstein relation, $\mathcal{T}_1 T \approx \mathcal{D}_F$, assuming that the coupling between the Brownian particle and the bath is weak [21].

Combining (10), (13) and (15), we find

$$\frac{\partial D}{\partial t} = \mathcal{A}\frac{\partial}{\partial p}(pD) + \mathcal{D}_F \frac{\partial^2 D}{\partial p^2} \ , \tag{16}$$

which describes the evolution of the momentum distribution of a test particle undergoing Brownian motion.

Next we proceed with solving the above equation with the boundary condition

$$D(p,t) \ \stackrel{t \to t_0}{\longrightarrow} \ \delta(p - p_0) \ . \tag{17}$$

The solution of (16) can be found by making a Fourier transform of $D(p,t)$,

$$D(p,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{D}(x,t)\ e^{ipx} dx \ , \tag{18}$$

where the inverse transform is

$$\tilde{D}(x,t) = \int_{-\infty}^{+\infty} D(p,t)\ e^{-ipx} dp \ . \tag{19}$$

Under the Fourier transform the corresponding initial condition follows from (17) and (19) as

$$\tilde{D}(x_0, t = t_0) = e^{-ip_0 x_0} \tag{20}$$

where $x = x_0$ at $t = t_0$ is assumed.

Replacing $p \rightarrow i\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial p} \rightarrow ix$, the Fourier transform of (16) becomes

$$\frac{\partial \tilde{D}}{\partial t} + \mathcal{A}x\frac{\partial \tilde{D}}{\partial x} = -\mathcal{D}_F\, x^2 \tilde{D} \,. \tag{21}$$

This is a first order partial differential equation which may be solved by the method of characteristics [23]. The characteristic equation corresponding to (21) reads

$$\frac{\partial t}{1} \ = \ \frac{\partial x}{\mathcal{A}x} \ = \ -\frac{\partial \tilde{D}}{\mathcal{D}_F x^2 \tilde{D}} \quad . \tag{22}$$

Along with the boundary condition in (20) and $\mathcal{A}(t_0) = \mathcal{D}_F(t_0) = 0$, the solution of (21) can be obtained as

$$\tilde{D}(x,t) \;=\; \exp\left[-i\,p_0\,x(t)\,e^{-\int^t \mathcal{A}(t')dt'}\right]\,\exp\left[-\int^t \mathcal{D}_F(t')\,x^2(t')\,dt'\right]\;, \tag{23}$$

Combining (18) and (23) yields

$$D(p,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp\left[-i\,p_0\,x(t)\,e^{-\int^t \mathcal{A}(t')dt'}\right]\,\exp\left[-\int^t \mathcal{D}_F(t')\,x^2(t')\,dt'\right]\,e^{ipx(t)}\,dx\,. \tag{24}$$

It is convenient to integrate over x_0 instead of x and to substitute the solution of the first pair of (22) into the above equation, leading to

$$\begin{aligned} D(p,t) = \frac{\exp\left(\int^t \mathcal{A}(t')\,dt'\right)}{2\pi}\int_{-\infty}^{+\infty} \exp\Bigg[&-i\,p_0\,x_0 \;+\; i\,px_0\,e^{\int^t \mathcal{A}(t')dt'} \\ &- x_0^2\left\{\int^t \mathcal{D}_F(t')\,\left(e^{2\int^{t'} \mathcal{A}(t'')\,dt''}\right)\,dt'\right\}\Bigg]\,dx_0\,. \end{aligned} \tag{25}$$

Using the standard form

$$\begin{aligned} f(z) \;&=\; \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp\left[-iuz \;+\; iuK_1 \;-\; \frac{1}{2}u^2K_2\right]\,du \\ &=\; \frac{1}{\sqrt{2\pi K_2}}\,\exp\left[-\frac{1}{2}(z-K_1)^2/K_2\right]\;, \end{aligned} \tag{26}$$

one can easily perform the integration in (25), resulting in

$$\begin{aligned} D(p,t) \;=&\; \frac{1}{\sqrt{\pi\,\left(4\int^t \mathcal{D}_F(t')\,\exp\left[2\int^{t'} \mathcal{A}(t'')\,dt''\right]\,dt'\right)\,\left[\exp\left(-2\int^{t'} \mathcal{A}(t'')\,dt''\right)\right]}} \\ &\times\,\exp\left[-\frac{\left(p - p_0\,e^{-\int^t \mathcal{A}(t')\,dt'}\right)^2}{\left(4\int^t \mathcal{D}_F(t')\,\exp\left[2\int^{t'} \mathcal{A}(t'')\,dt''\right]\,dt'\right)\,\left[\exp\left(-2\int^{t'} \mathcal{A}(t'')\,dt''\right)\right]}\right]\,. \end{aligned} \tag{27}$$

For relativistic particles, $p = E$, (27) can be written as

$$D(E,L) = \frac{1}{\sqrt{\pi\,\mathcal{W}(L)}}\,\exp\left[-\frac{\left(E - E_0\,e^{-\int_0^L \mathcal{A}(t')\,dt'}\right)^2}{\mathcal{W}(L)}\right]\;, \tag{28}$$

where $\mathcal{W}(L)$ is given by

$$\mathcal{W}(L) = \left(4\int_0^L \mathcal{D}_F(t')\exp\left[2\int^{t'} \mathcal{A}(t'')\,dt''\right]\,dt'\right)\,\left[\exp\left(-2\int_0^L \mathcal{A}(t')\,dt'\right)\right]\;. \tag{29}$$

The energy loss of partons in the QGP due to elastic collisions was estimated in Ref. [12] and we used the expression averaged over parton species as

$$-\frac{dE}{dL} \;=\; \frac{4}{3}\left(1+\frac{9}{4}\right)\,\pi\alpha_s^2T^2\,\left(1+\frac{n_f}{6}\right)\log\left[2^{n_f/2(6+n_f)}\,0.92\,\frac{\sqrt{ET}}{m_g}\right]\;, \tag{30}$$

where n_f is the number of quark flavours, α_s is the strong coupling constant, $m_g = \sqrt{(1+n_f/6)gT/3}$ is the thermal gluon mass, and E is the energy of the partons. Following (14), we can now estimate $\mathcal{A}$ for different quark flavours and gluons at the energies (temperatures) of interest. For averaging over the momentum the Boltzmann distribution was used. The time dependence of the drag coefficient comes from assuming a temperature $T(t)$ decreasing with time as

the system expands, according to the Bjorken scaling law [24] $T(t) = t_0^{1/3} T_0/t^{1/3}$, where t_0 and T_0 are, respectively, the initial time and temperature at which the background of the partonic system has attained local kinetic equilibrium. Since the plasma expands with the passage of time, we have used the length of the plasma, L as the maximum time limit for the relativistic case ($v \sim 1$). In Fig. 1 the drag coefficient in the QGP phase of the expanding fireball is shown as a function of time, where we have chosen the parameters $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm, and $\alpha_s = 0.3$.

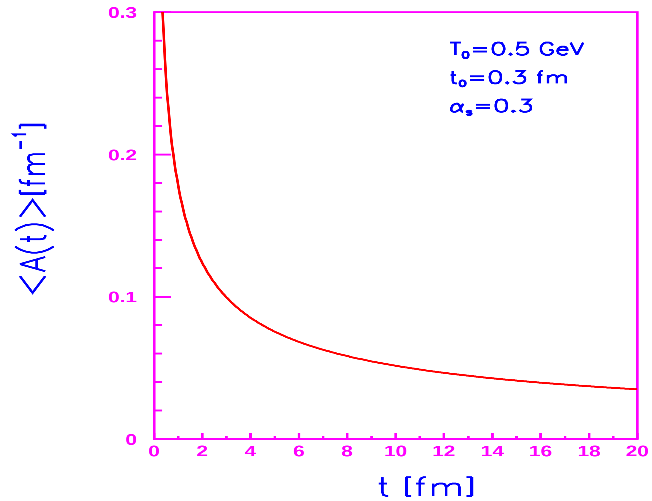

Figure 1: The drag coefficient $\mathcal{A}(t)$ in an expanding QGP.

In Fig. 2 we show the probability distribution, $D(E, L)$ given in (28), as a function of energy E, choosing $E_0 = 1$ GeV. The peak of the probability distribution is shifted with passage of time (or distance travelled) indicating the most probable energy loss due to elastic collisions in the medium.

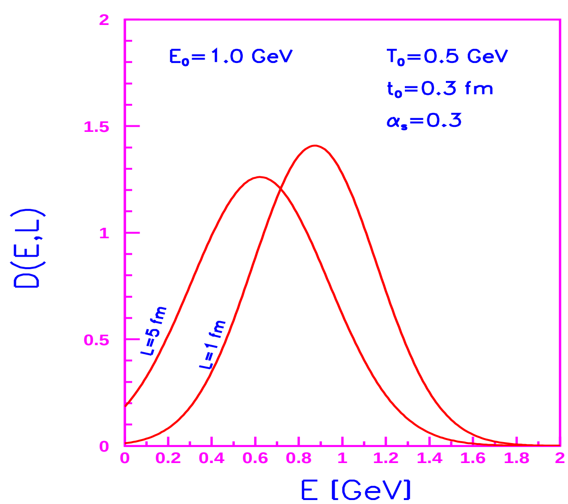

Figure 2: The energy-loss probability distribution, $D(E, L)$ as a function of energy E for a given distance L.

After traversing a distance L, the mean energy of the parton due to the elastic collisions in the medium can be estimated as

$$\langle E \rangle = \int_0^\infty E\, D(E, L)\, dE = E_0\, e^{-\int_0^L \mathcal{A}(t')\, dt'} = m_\perp\, e^{-\int_0^L \mathcal{A}(t')\, dt'}, \tag{31}$$

where $E_0 = m_\perp = \sqrt{p_\perp^2 + m^2}$ at the central rapidity region, $y = 0$. The average energy loss due to elastic collisions in the medium is given by

$$\Delta E = \langle \epsilon \rangle = E_0 - \langle E \rangle$$

$$= m_\perp \left(1 - e^{-\int_0^L \mathcal{A}(t')\, dt'} \right) . \tag{32}$$

For the massless case, the average $\Delta p_\perp$ can be written as

$$\Delta p_\perp \;=\; p_\perp \left(1 - e^{-\int_0^L \mathcal{A}(t')\, dt'} \right) . \tag{33}$$

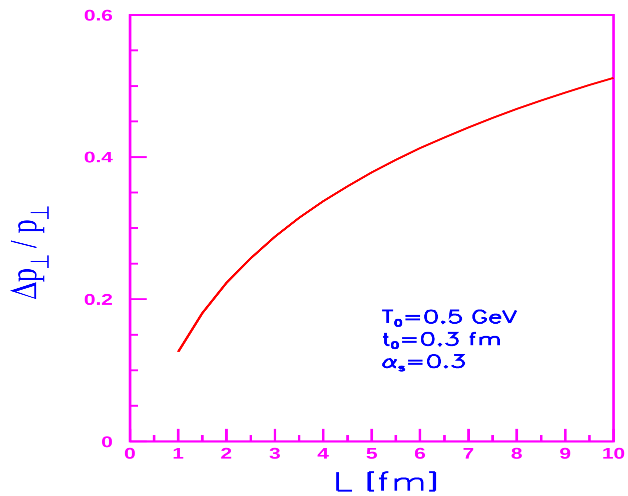

Figure 3: The effective shift of the scaled transverse momentum $\Delta p_\perp / p_\perp$ as a function of distance L.

In Fig. 3 we display the scaled effective energy loss of fast partons due to elastic collisions in the medium as function of the distance L. It is found that the effective energy loss, defined as the shift of the momentum spectrum of fast partons, depends on the transverse momentum $p_\perp$ and the distance L. For given $p_\perp$ the effective energy loss is 10% after traversing a distance of 1fm and around 50% after 10 fm. The effective energy-loss scales linearly with $p_\perp$ for a given L. This clearly reflects a random walk in $p_\perp$ as a fast parton moves in the medium [19] with some interactions resulting in an energy gain and others in a loss of energy.

We assume that the geometry is described by a cylinder of radius R, as in the Boost invariant Bjorken model [24] of nuclear collisions, and the parton moves in the transverse plane in the local rest frame. Then a parton created at a point $\vec{\mathbf{r}}$ with an angle ϕ in the transverse direction will travel a distance [19]

$$L(\phi) = (R^2 \,-\, r^2 \,\sin^2 \phi\,)^{1/2} \,-\, r\, \cos \phi \;\; , \tag{34}$$

where $\cos \phi \,=\, \hat{\vec{\mathbf{v}}} \cdot \hat{\vec{\mathbf{r}}}$; $\vec{\mathbf{v}}$ is the velocity of the parton and $r \,=\, |\vec{\mathbf{r}}|$.

The parameterization of the $p_\perp$ distribution [19,25] which describes the first RHIC hadroproduction data for moderately large values of $p_\perp$ has the form

$$\frac{dN^{\rm vac}}{d^2 p_\perp} \;=\; N_0 \left(1 \,+\, \frac{p_\perp}{p_0} \right)^{-\nu} \;\; , \tag{35}$$

where $\nu \approx 8$ and $p_0 \,=\, 1.75$ GeV. The quenched spectrum convoluted with the transverse geometry of the partonic system can be written from (1) as

$$\frac{dN^{\rm med}}{d^2 p_\perp} \;=\; Q(p_\perp)\, \frac{dN^{\rm vac}}{d^2 p_\perp} \;=\; \frac{1}{2\pi^2 R^2} \int_0^{2\pi} d\phi \int_0^R d^2 r\, \frac{dN(p_\perp + \Delta p_\perp)}{d^2 p_\perp} \;\; . \tag{36}$$

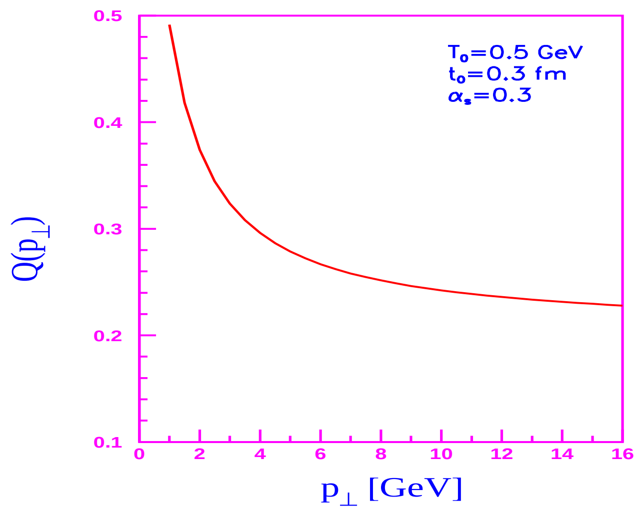

Figure 4: The quenching factor $Q(p_\perp)$ as a function of transverse momentum $p_\perp$.

In Fig. 4 our numerical results of the quenching factor $Q(p_\perp)$ due to elastic collisions in the medium and its dependence on $p_\perp$ are shown. It is found that the quenching becomes stronger at lower $p_\perp$ and gradually decreases with higher $p_\perp$ in agreement with experiments [17]. As discussed above the quenching factor obtained here is quite similar to a random walk as well as the Bethe-Heitler scaling law given phenomenologically in Ref. [19].

Comparing the quenching factor due to elastic collisions, shown in Fig. 3, with the one coming from the radiative energy loss [19] we observe that both are of similar magnitude. The radiative quenching factor varies for $p_\perp$ from 6 to 16 GeV between 0.17 and 0.24 depending on the model assumptions, whereas the collisional quenching factor lies between 0.22 and 0.26 for the same momentum range and for our choice of the parameters, i.e., $T_0 = 0.5$ GeV, $t_0 = 0.3$ fm, and $\alpha_s = 0.3$. Apart from uncertainties in these parameters let us also have a look at some of the assumptions made in this work which may modify the quenching factor. First, as discussed above, the momentum dependence of the drag coefficent, containing the dynamics of the elastic collisions, has been averaged out. A major advantage of this is the simplicity of the resulting differential equation. Of course, this simplification can lead to some amount of uncertainty in the quenching factor. Secondly, the entire discussion is based on the one dimensional Fokker-Planck equation and the Bjorken model of the nuclear collision, which may not be a very realistic description here but can provide a very intuitive picture of the problem. However, extension to three dimension is indeed an ambitious goal, which may cause that many of the considerations of the present work will have to be revised. Within the limitation of our simplified model the contribution of the collisional energy loss of partons in the QGP to the quenching of hadron spectra in ultrarelativistic heavy-ion collisions is found to be very important and cannot be neglected. In particular, if the radiative energy loss is suppressed at intermediate energies, the collisional quenching would explain the observed decreasing of the quenching factor with increasing transverse momentum.

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